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## Grade 10 Printable Sample Problems Package

Enclosed are the following sample problems which have been designed to show real-world technical applications of key concepts covered in *Foundations of Mathematics and Pre-Calculus 10*:

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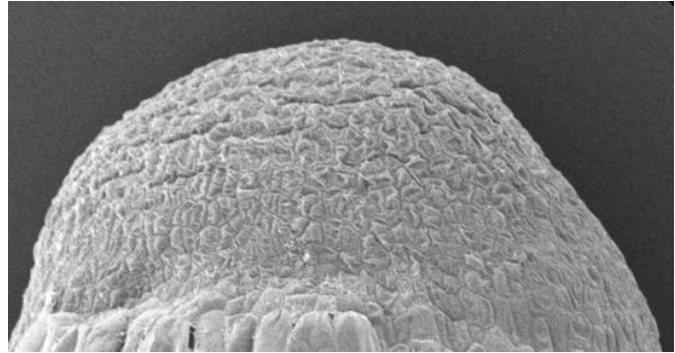
We hope these problems will be useful.

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## Plant Tip Growth

### Introduction:

Many parts of plant growth occur through **simple tip growth**, a process by which the plant elongates a cylindrical shaft with an approximately hemispherical tip. For instance, the illustration at right shows the side view of a larch tree embryo.



Larch tree embryo<sup>1</sup>

Modeling the tip as a **hemisphere** with a diameter of **0.728 millimeters**, determine the following.

### Problems: You Try It!

#### PART A

Molecules that adhere to the surface of the plant will determine how it grows, so the surface area of the tip of a plant affects the growth rate of the plant. Find the **surface area** of the hemispherical part at the tip of the larch tree embryo shown above.

Express your answer in square millimeters, correct to three significant figures.

#### PART B

The concentration of molecules inside the tip of a plant affects the growth of the plant (see below for more information). Find the **volume** of the hemispherical tip described above, a first step towards calculating the concentration of molecules inside the tip of the plant.

Express your answer in cubic millimeters, correct to three significant figures.

#### PART C

The amount of nutrients required by a plant will depend on how much tip tissue there is (that is, the mass of the tip tissue). If the tissue has a density of 0.771 grams per cubic centimeter (that is, 1  $\text{cm}^3$  of material has a mass of 0.771 g). What is the mass of the hemispherical tip?

Express your answer in grams, correct to three significant figures.

#### PART D

If 45% of the tip is cellulose by mass, what is the mass of cellulose in the tip?

Express your answer in grams, correct to three significant figures.

<sup>1</sup> Patrick von Aderkas. Larch tree embryo. No date. Personal collection. 17 August 2012.

**Additional Background Information:**

**PART B**

The concentration of molecules inside the tip of a plant will also affect how the plant grows. If it is fairly uniform, the concentration of molecules inside the tip can be calculated by dividing the number of molecules by the volume. Molecules are usually counted in very large groups called moles, in much the same way that eggs are usually counted in groups called dozens.



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**Answers:**

A.  $0.832 \text{ mm}^2$

B.  $0.101 \text{ mm}^3$

C.  $m = 7.79 \times 10^{-5} \text{ grams}$

D.  $3.50 \times 10^{-5} \text{ grams}$

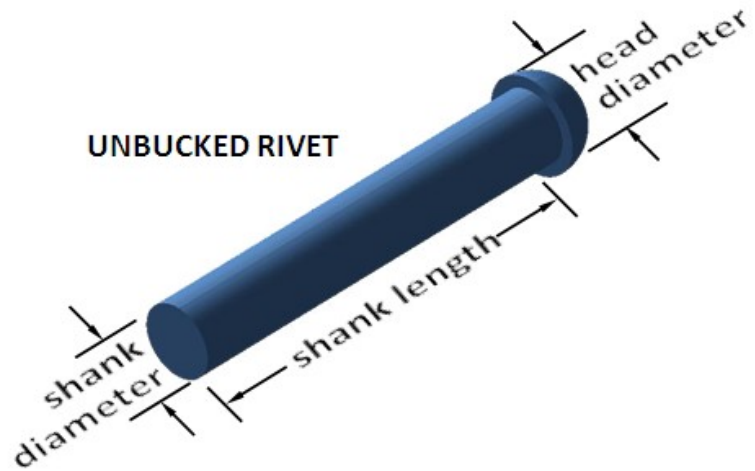


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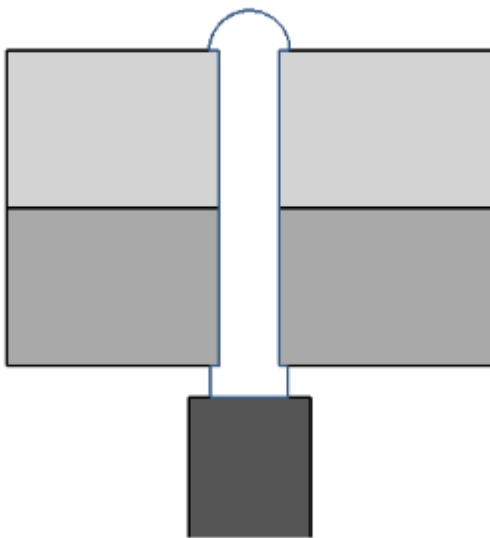
## Rivets

### Introduction:

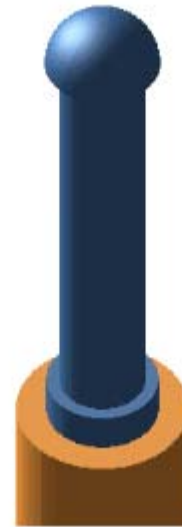
A rivet (shown below) is used to assemble metal components together. Before it is slotted through the holes in the metal it is intended to hold together, the rivet is referred to as "**unbucked**". The unbucked rivet consists of a hemispherical head with diameter 0.0314 inches, attached to a cylindrical shank with a diameter 0.09045 inches and length 1.19 inches.



Once the rivet has been pushed through the metal it is intended to fasten, the rivet is secured in place by flattening the end of the shank farthest from the hemispherical head.

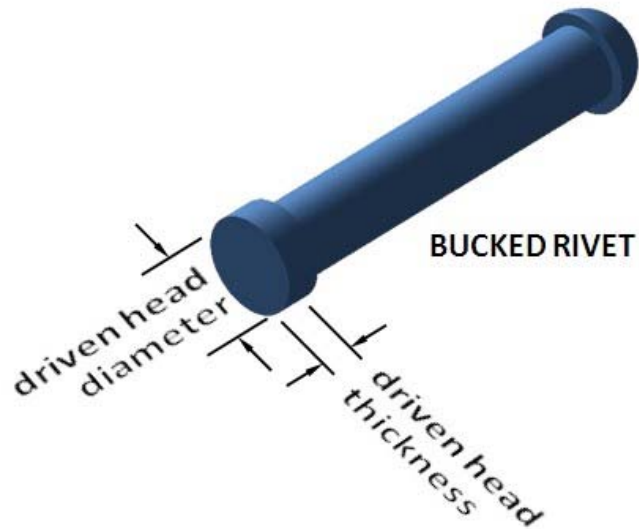


A two-dimensional side view of the rivet being slotted through the holes drilled in two pieces of metal that are to be fastened together. The end of the shank is then flattened to form the **driven head**.



A three-dimensional view of the same rivet as it is slotted through the holes in the metal (not shown) and the end of the shank is flattened to form the **driven head**.

This flattened end, in the shape of a cylinder that is now wider than the unbucked shank, is then referred to as the "driven head" (which is different from the hemispherical head at the other end).



The driven head must satisfy certain safety specifications in order for the rivet to be considered securely fastened. For the unbucked rivet described above, the **driven head diameter** (the diameter across the flattened part of the shank) must be **at least** 0.122 inches and the **driven head thickness** must be a **minimum** of 0.038 inches and a **maximum** of 0.05 inches.

### Problems: You Try It!

Follow the steps below to determine the **maximum** thickness of metal that the rivet described can be used to fasten together.

#### PART A

What is the volume of the shank of the unbucked rivet?

Express your answer in cubic inches, correct to three significant figures.

#### PART B

Assume that the volume of the rivet does not change as a result of the driven head being flattened. If the thickness of metal being fastened is at its **maximum**, should the thickness of the driven head be at its **maximum** value, its **minimum** value, or something in between?

#### PART C

Find the appropriate volume of the driven head, based on your answer in Part B.

Express your answer in cubic inches, correct to three significant figures.

#### PART D

Knowing that the volume of the rivet does not change, and based on your answers in Part A and Part C, what is the **maximum** thickness of metal that the rivet described can be used to fasten together?

Express your answer in inches, correct three significant figures.

**Additional Background Information:**

The illustration at right shows a rivet, as might be used, for instance, in assembling machinery or in bridge construction. Rivets are driven through holes drilled in metal components and fixed in place in order to fasten the components firmly together and in tight alignment.



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**Answers:**

- A.  $0.00765 \text{ in}^3$
- B. Minimum driven head thickness
- C.  $0.000444 \text{ in}^3$
- D. 1.12 in



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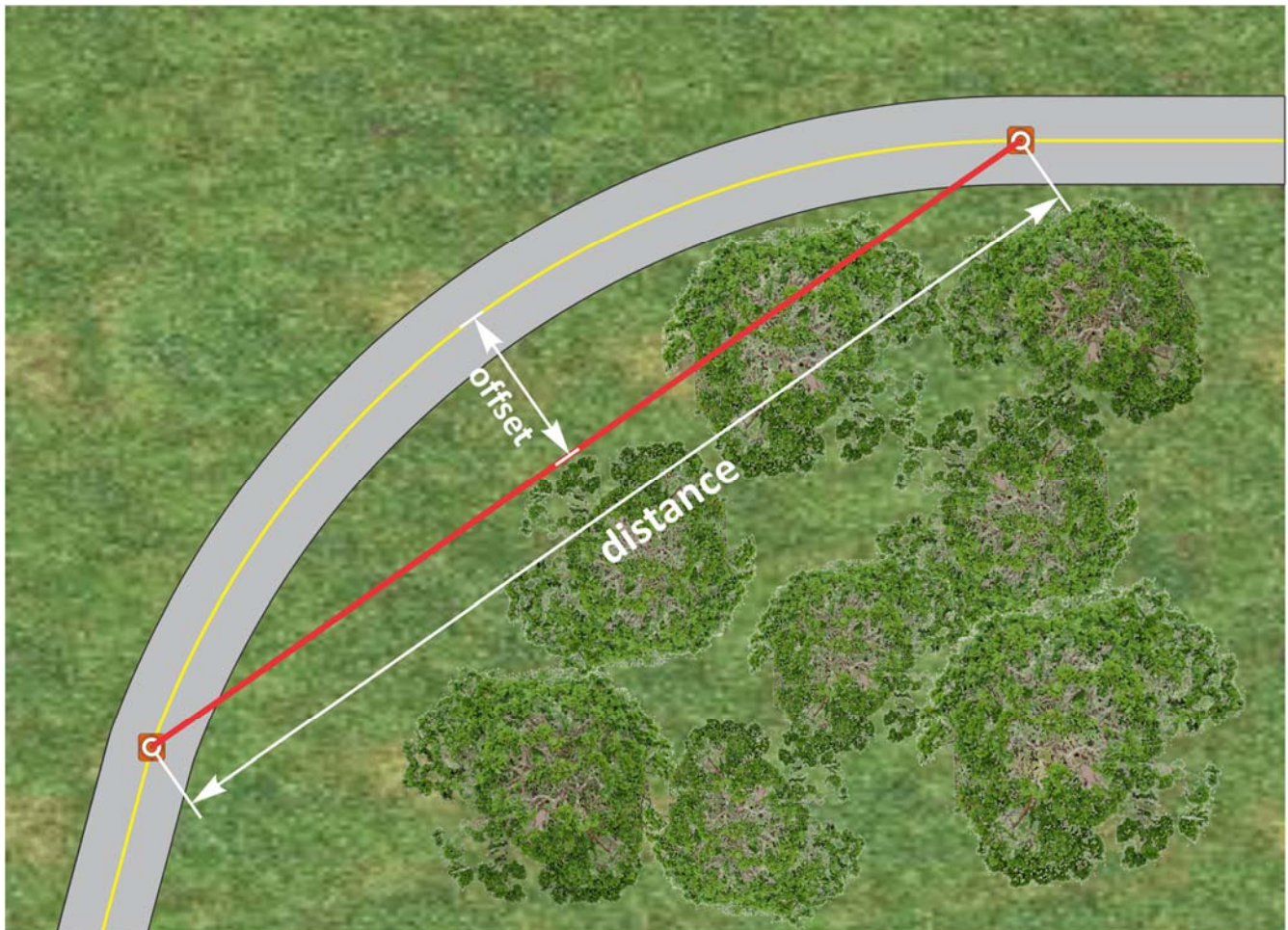


## Curved Road

### Introduction:

One curve on a two-lane highway has had six collisions on it in the last two years, with one fatality and two injuries. The roads engineer wonders whether the curve may be too sharp for the speed limit of 90 km/h. If so, he should install a warning sign with a lower advisory speed on it to reduce accidents.

At the site, he measured a distance of 45 m between the two ends of the curve, at the center line of the road (red line in the diagram below). The distance from the midpoint of the measurement to the center line of the road was 2.40 m (labelled as "offset" in the diagram below).

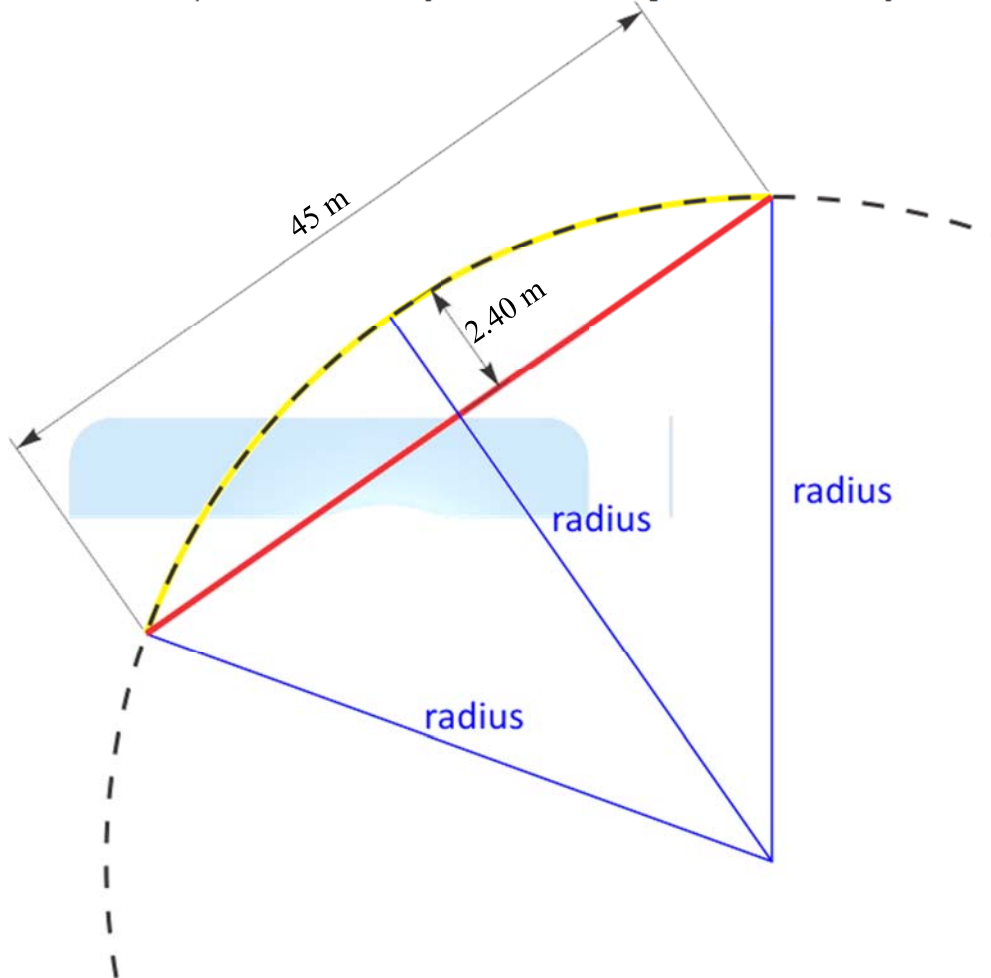


## Problems: You Try It!

### PART A

Assuming that the curve is a circular arc, what is the **radius** of the curve?

*Hint:* You may find that the diagram below is helpful in determining how to calculate this.



Express your answer in meters, correct to three significant figures.

### PART B

Lateral (or radial) acceleration for a vehicle driving at a speed of  $V$  m/s along a curve with a radius of  $r$  meters can be calculated with the formula:

$$A_r = \frac{V^2}{r}$$

If the maximum safe lateral acceleration for normal driving in wet conditions is 0.215  $g$  (where  $g$  is the unit for acceleration due to gravity), what is the highest safe cornering speed on this curve?

You may require the following unit conversion factors to solve this problem:

$1\text{ g} = 9.806\text{ m/s}^2$	$1\text{ km} = 1000\text{ m}$	$1\text{ h} = 3600\text{ s}$
-----------------------------------	-------------------------------	------------------------------

Express your answer in kilometers per hour, correct to three significant figures.

### PART C

If the maximum safe speed is less than the posted speed limit, a warning sign should be posted. Should such a sign be posted?

### PART D

The advisory curve speed on the sign must be **rounded down to the nearest multiple of 10 km/h**. What advisory speed should be posted?

Express your answer in kilometers per hour.



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**Answers:**

- A. 107 m
- B. 54.0 km/h
- C. yes
- D. 50 km/h



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## Marina Ramp

### Introduction:

At a Vancouver marina, a pedestrian ramp leads to a floating dock which moves up and down with the tide. The upper end of the ramp is hinged (it can rotate) and the lower end is supported by a roller that can roll back and forth along a track (see Additional Background Information for labelled diagram). As the tide rises and falls, the slope of the ramp changes, and the roller on the lower end moves in its track. **Note: Diagrams are not to scale.**



The ramp can't be too steep, since users of the marina must **safely** carry equipment up and down the ramp. The slope of the ramp **must not exceed 39%**. The slope gives the relationship between the vertical and horizontal distances covered by the ramp (i.e. that the vertical drop can be **at most 39%** of the horizontal distance travelled). The difference in elevation between the lowest low tide and the highest high tide at this location is 4.3 m.



## Problems: You Try It!

### PART A

Assuming that the ramp is **horizontal** when the water level is at its highest (as shown above), determine the minimum length of the ramp such that its slope will never exceed 39%.

Express your answer in meters, correct to **2 decimals**.

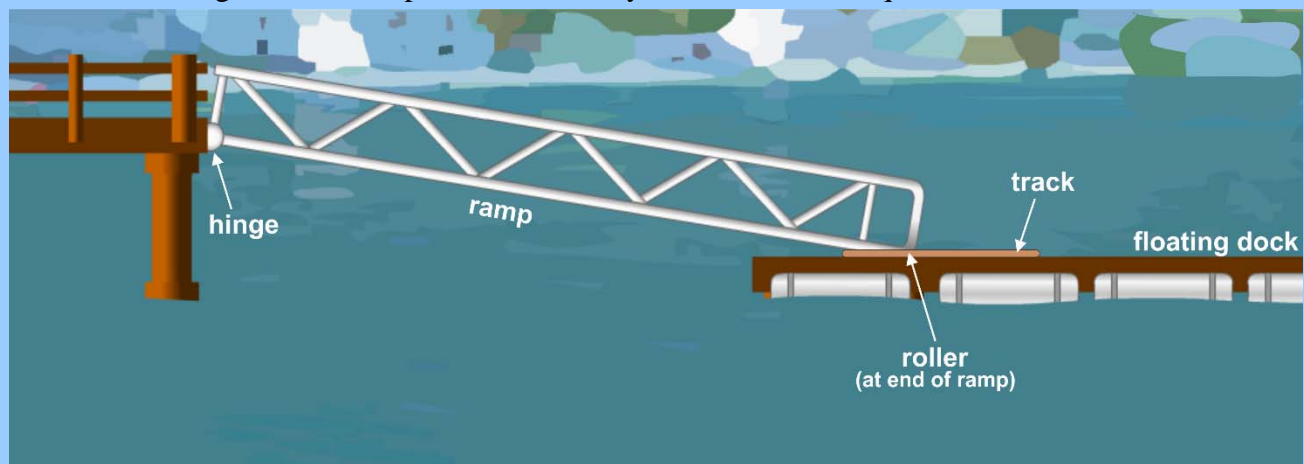
### PART B

Using the minimum ramp length calculated above, determine the minimum length of the roller track (in mm) required to support the lower end of the ramp.

Express your answer in millimeters, correct to significant figures.

#### Additional Background Information:

The labelled diagram below is provided to clarify terms used in the question.



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**Answers:**

A. 11.83 m

B. 809 mm



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## Atmospheric Pressure and Altitude

### Introduction:

Atmospheric pressure is a measure of pressure due to the mass of the air in the atmosphere. As an airplane takes off, the atmospheric pressure on the outside of the airplane **decreases** as there is less air above it as it rises.

Atmospheric pressure at elevations above sea level can be estimated as:

$$p = 101325 \left( 1 - (2.256 \times 10^{-5})h \right)^{5.25578}$$

where:

- $p$  is the atmospheric pressure in **pascals (Pa)**, and
- $h$  is the elevation above sea level in **meters (m)**.

### Problems: You Try It!

#### PART A

Calculate the atmospheric pressure at an elevation of 9,500 m above sea level.

Express your answer in pascals, correct to correct to three significant figures.

#### PART B

What would the atmospheric pressure be at an elevation of 0.5 **km** above sea level.

Express your answer in **kilopascals (kPa)**, correct to three significant figures.

**Useful unit conversion: 1 kPa = 1000 Pa**

#### PART C

The altitude of an airplane can be found by measuring the pressure on the airplane. Use any appropriate method and the equation given above to estimate or calculate the altitude of an airplane when the atmospheric pressure outside of the airplane is 42,000 Pa.

Express your answer in meters, correct to the nearest **500 m**.

<sup>1</sup> Alan Sim. *Aircraft altimeter*. 3 August 2005. *Wikimedia Commons*. Web. 19 May 2015.



**Additional Background Information:**

Atmospheric pressure is used to calculate the altitude of an airplane (the height above sea level). Altimeters are able to display the correct altitude by measuring the pressure outside of the airplane.

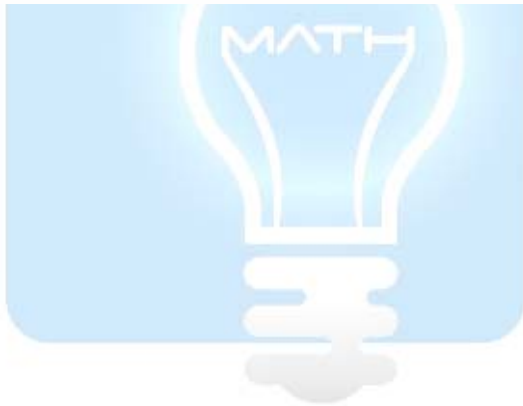


Kollsman-type altimeter<sup>1</sup>



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**Answers:**

A. 28500 Pa

B. 95.5 kPa

C. 7000 m



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## Mars Rover Solar Power

### Introduction:

As humanity pushes the boundaries of space exploration, the power that can be obtained from the Sun's energy at different locations in the solar system becomes increasingly important to understand.

For Mars Rovers powered by solar cells, the amount of power they can collect from the Sun's energy affects the information they can collect and transmit back to Earth.



Artist's Rendering of a Mars Exploration Rover<sup>1</sup>

### PART A

The Sun's radiation can be modeled by a black body (see below for more information) radiating at a temperature of 5,800 K. The radiation intensity at the surface of the Sun is the power emitted per unit area of Sun surface. The radiation intensity  $\phi$  in watts per square meter is:

$$\phi = \sigma T^4$$

where  $\sigma$  is the Stefan-Boltzmann constant ( $5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$ ), and T is the temperature in kelvin (K). Calculate the **radiation intensity at the surface of the Sun** in  $\text{W/m}^2$  (watts per square meter).

Express your answer in watts per square meter correct to three significant figures

### PART B

To calculate the total power radiated by the Sun, we need the surface area of the Sun. The Sun may be approximated as a sphere with a radius of  $7 \times 10^8 \text{ m}$ . What is the **surface area of the Sun**?

Express your answer in square meters, correct to three significant figures, using scientific notation if needed.

### PART C

Based on the radiation intensity (power per square meter of solar surface) and the surface area of the Sun, calculate the **total power generated by the Sun**.

Express your answer in watts, correct to three significant figures, using scientific notation if needed.

<sup>1</sup> De Wood, Pooley, USDA, ARS, EMU. "Campylobacter bacteria are the number-one cause of bacterial food-related gastrointestinal illness in the United States." 2 January 2008. *Wikimedia Commons*. Web. 24 June 2013.

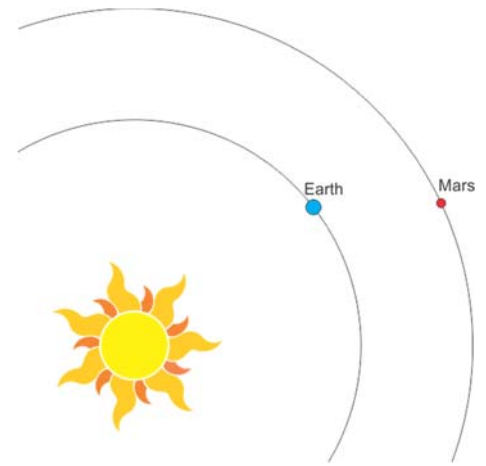
### More Information:

The intensity of the solar radiation that reaches a body in the solar system decreases as the distance from the Sun increases. At a particular location in the solar system,

$$\text{Radiation Intensity} = \frac{P_{\text{sun}}}{A_{\text{sun}}}$$

where:

- $P_{\text{sun}}$  is the total power generated by the Sun (from Part C), and
- $A_{\text{sphere}}$  is the surface area of a sphere with a radius equal to the distance from the Sun to that location.



Sketch of Earth and Mars orbits, not to scale

### PART D

Shortly after a Rover leaves Earth, it is a distance of  $1.66 \times 10^{11}$  m from the Sun. What is the **intensity of the Sun's radiation** at this point in its journey?

Express your answer in  $\text{W/m}^2$ , correct to three significant figures.

### PART E

Just before the Rover arrives at Mars, it is a distance of  $2.18 \times 10^{11}$  m from the Sun. What is the **intensity of the Sun's radiation** at this point in its journey?

Express your answer in  $\text{W/m}^2$ , correct to three significant figures.

### PART F

By what percentage does the intensity of the Sun's radiation **decrease** as the distance from the Sun increases from  $1.66 \times 10^{11}$  m to  $2.18 \times 10^{11}$  m?

Express your answer in percent, correct to three significant figures.

#### Additional Background Information:

A black body is an idealized concept in physics which can be used to determine the electromagnetic radiation from an object or "body" based only on a temperature.

In many technical applications, temperatures are used in units of kelvin (K). It is related to the more familiar degrees Celsius system by **adding** 273.15 to get from  $^{\circ}\text{C}$  to K (so  $0^{\circ}\text{C} = 273.15$  K).

NASA's Mars Exploration Rovers use solar panels to generate power. During their journey from Earth to Mars, the amount of power generated by the solar panels decreases significantly, by approximately 50%.



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**Answers:**

- A.  $6.42 \times 10^7 \text{ W/m}^2$
- B.  $6.16 \times 10^{18} \text{ m}^2$
- C.  $3.95 \times 10^{26} \text{ W}$
- D.  $1.14 \times 10^3 \text{ W/m}^2$
- E.  $662 \text{ W/m}^2$
- F. 42.0%



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## Bacteria in Food

### Introduction:

Researchers in the field of food safety develop mathematical models which attempt to predict bacteria growth in food. These models can help government agencies (such as Health Canada) develop guidelines for the safe handling and storage of food.

### Problems: You Try It!

#### PART A

The number of a specific type of bacteria in a certain food can be approximated using the following formula:

$$N = -T^2 + 72T - 396$$

where  $N$  is the number of bacteria, and

$T$  is the temperature of the food in degrees Celsius.

One can determine the temperature(s) at which the number of bacteria is negligible by solving the following equation for  $T$ :

$$T^2 - 72T + 396 = 0$$

Perform the **first step** in solving the equation above by factoring the left-hand side of the equation.

#### PART B

For a different food product, the factored form of the equation relating the number of bacteria to the temperature is given by:

$$N = -(T - 2)(T - 57)$$

Multiply the factors on the right-hand side of the equation and write this expression in **expanded form**.



Campylobacter bacteria<sup>1</sup>

#### Additional Background Information:

Both formulas given in this problem are based on observations and experience rather than scientific or mathematical theories. Because of this, they are called **empirical equations**. The first empirical equation is only valid for temperatures ranging from 2°C to 70°C (because that is the range of temperatures observed in developing the equation).



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<sup>1</sup> De Wood, Pooley, USDA, ARS, EMU. "Campylobacter bacteria are the number-one cause of bacterial food-related gastrointestinal illness in the United States." 2 January 2008. *Wikimedia Commons*. Web. 24 June 2013.

**Answers:**

A.  $(T - 6)(T - 66)$

B.  $-T^2 + 59T - 114$



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## Blood Pressure Change

### Introduction:

Many types of medications are used to reduce high blood pressure. Health care professionals sometimes have access to mathematical models which have been developed in a research setting to establish a relationship between two or more variables, such as dosage (amount of medication given to a patient) and blood pressure change.



Various Medications<sup>1</sup>

### Problems: You Try It!

#### PART A

For a certain dosage of a drug,  $x$  (in mg), the resulting blood pressure change  $B$  (in mmHg) is approximated by the polynomial function:

$$B = 0.63x^2 - 0.35x^3$$

A medical professional wants to know the dosage at which there is no blood pressure change. She could solve the equation above for  $x$  when  $B = 0$ , that is:

$$0 = 0.63x^2 - 0.35x^3$$

Perform the **first step** in solving this problem by **factoring** the right-hand side of the equation.

**Note:** Since the numerical quantities given here are decimal values, follow the convention of leaving any **numerical factors** that may be common between terms **unfactored**.

#### PART B

The medical professional also wants to determine the dosage that will maximize the blood pressure change. By applying basic principles of calculus, she finds that the drug dosage at which the blood pressure change,  $B$ , will be a minimum or a maximum satisfies the following equation:

$$0 = 1.26x - 1.05x^2$$

Perform the **first step** in solving this problem by **factoring** the right-hand side of the equation.

**Note:** Since the numerical quantities given here are decimal values, follow the convention of leaving any **numerical factors** that may be common between terms **unfactored**.

<sup>1</sup> LadyofProcrastination. "An assortment of drugs, including 150mg Effexor XR (by Wyeth Pharmaceuticals), 10mg dicyclomine (by Watson), 100 mg sertraline (generic), 25 mg Topamax (by McNeil), and 10 mg amitriptyline (generic) in addition to vitamin E gelcaps and some generic ibuprofen gelcaps." 20 February 2008. *Wikimedia Commons*. Web. 24 June 2013.



## PART C

For a different type of blood pressure medication, the factored form of the equation relating blood pressure change  $B$  and drug dosage  $x$  is given by the following equation:

$$B = x^2(0.25 - 0.12x)$$

Multiply the monomial by the binomial in order to express the right-hand side of this equation in expanded form.



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**Answers:**

A.  $x^2(0.62 - 0.35x)$

B.  $x(1.26 - 1.05x)$

C.  $0.25x^2 - 0.12x^3$

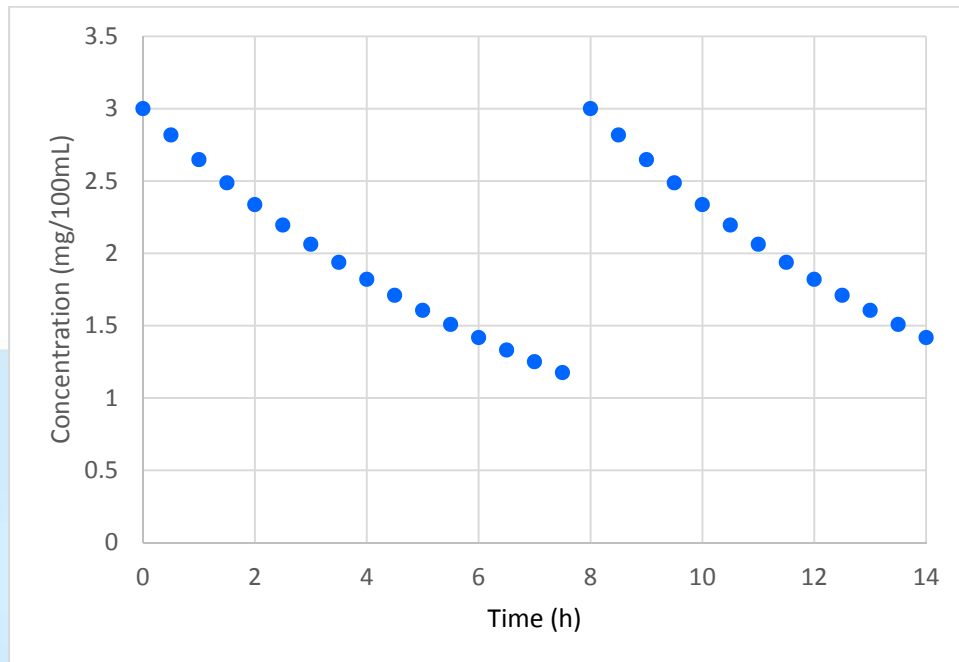


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## Penicillin Concentration

### Introduction:

When penicillin is administered to a patient, the concentration of the drug in the patient's veins (given in units of mg of penicillin per 100 mL of blood) is measured every 30 minutes. The results are displayed in the graph below. Each blue point represents one reading of the concentration of penicillin in the patient's blood stream. The first dose of penicillin is given at a time of zero hours (h).



### Problems: You Try It!

#### PART A

What is the concentration of penicillin in the patient's blood after 5 hours?

Express your answer in milligrams per 100 milliliters of blood, **correct to one decimal**. Do **not** include units in your answer.

#### PART B

This patient is receiving penicillin doses at regular time intervals. How much time passes between doses?

Express your answer in hours, correct to the nearest **whole number**. Do **not** include units in your answer.

#### PART C

In between doses, is the relationship between time and concentration **linear**?

**Additional Background Information:**

In setting a dosing schedule for a particular medication, health care professionals may wish to ensure that the level of the medication in the bloodstream does not drop below a certain level. To assess this, they can regularly monitor the concentration of the drug over time to ensure set criteria are being met.



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**Answers:**

- A. 1.6 mg/100mL
- B. 8 hours
- C. No, it is not linear



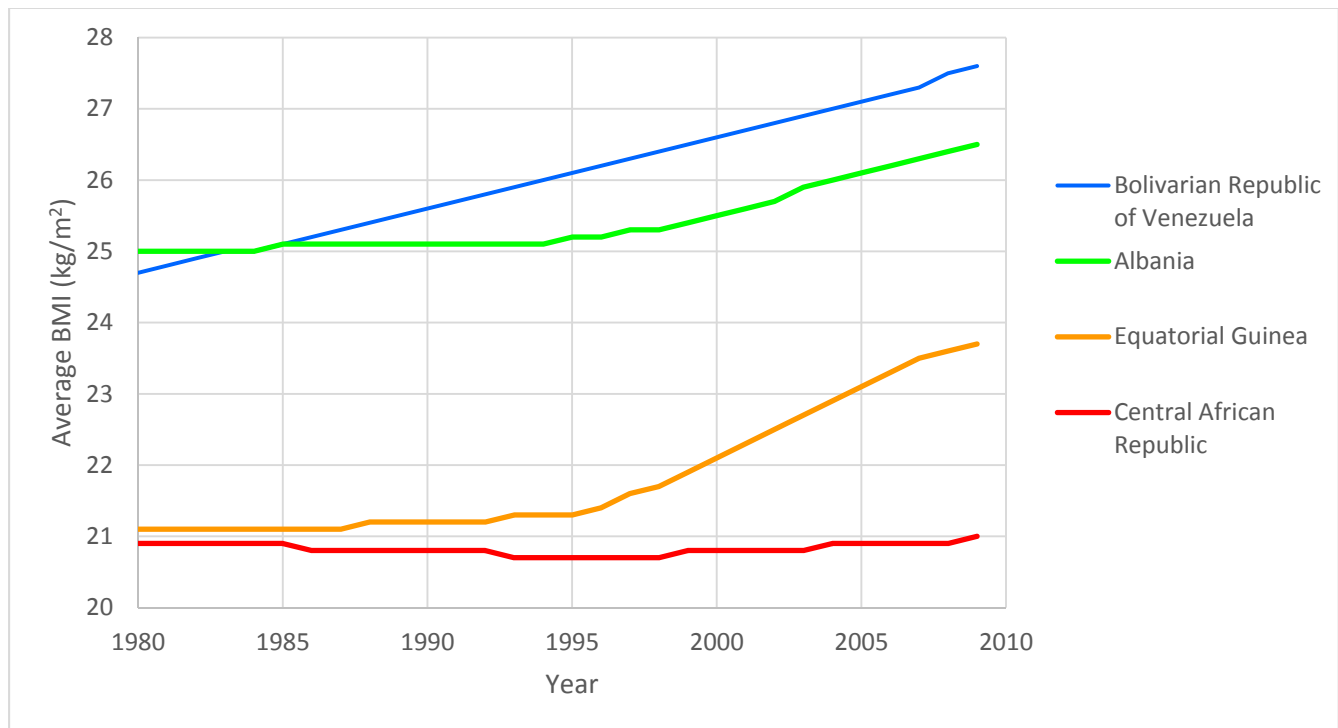
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## World Health Body Mass Index

### Introduction:

The World Health Organization (WHO) is responsible for many worldwide health initiatives, including monitoring health-related traits. One characteristic that the WHO tracks is the average body mass index (BMI) for a country (measured in kilograms per square meter).

The average BMI of Bolivarian Republic of Venezuela, Albania, Equatorial Guinea, and Central African Republic from 1980 to 2009 is shown in the graph below.



### Problems: You Try It!

#### PART A

Which of the four countries shown had the **highest average BMI** in 2009?

#### PART B

Which country, if any, had an average BMI that **decreased** from 1980 to 2009?

#### PART C

Comparing the graphs for Albania and Central African Republic, which country tended to have a **lower** average BMI during the years 1980 to 2009?

#### PART D

What is the **average rate of change** for the BMI of Equatorial Guinea from 1980 to 2009? This is the same as the slope of a line drawn between the average BMI in 1980 and the average BMI in 2009.

Express your answer in kilograms per square meter per year, correct to at least 2 decimals. Do **not** include units in your answer.

## PART E

Comparing the graphs for Bolivarian Republic of Venezuela and Albania, which country had an average BMI that **increased more quickly** (i.e. had a higher average rate of change) from 1980 to 2009?

### Additional Background Information:

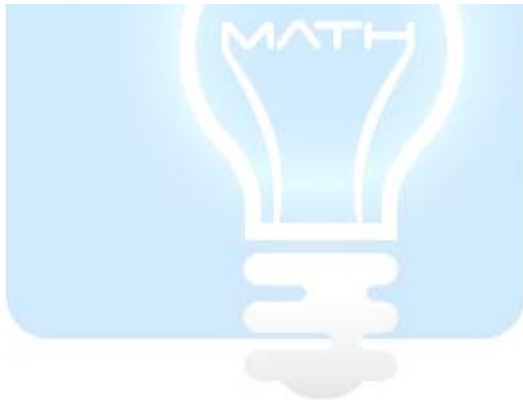
The World Health Organization (WHO) is a part of the United Nations, and is responsible for many worldwide health initiatives. As a part of their mandate, they monitor and assess health trends around the world, and much of this data is available through the WHO website under "Data and statistics".

Among many other variables, they track risk factors for noncommunicable diseases such as heart disease, stroke, and diabetes. The WHO tracks body mass index trends as BMI is considered to be a risk factor for many noncommunicable diseases.



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**Answers:**

- A. Bolivarian Republic of Venezuela
- B. None
- C. Central African Republic
- D.  $0.09 \frac{\text{kg/m}^2}{\text{year}}$
- E. Bolivarian Republic of Venezuela



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## Bronze Alloy

### Introduction:

A historical reenactor is trying to fabricate a replica of a bronze sword discovered on an archeological site. Attempting to replicate such artifacts using similar methods can help historians and archeologists better understand the processes used in the past.



Bronze Age Sword<sup>1</sup>

The sword found on the archeological site contains **10.1% tin**. The reenactor has two modern sources of bronze which he plans to combine to fabricate the replica:

- an alloy that is **17% tin** and
- an alloy that is **9% tin**.

He plans to produce **3.06 kg** of an alloy that is **10.1% tin**, and wants to know how much of each of the two modern alloys he needs to use to make the desired alloy.

All percentages in the alloys are given by mass.

### Problems: You Try It!

For one particular piece of copper wire, the resistance is  $13.2 \Omega$  at a temperature of  $14^\circ\text{C}$ . When the temperature is increased to  $27^\circ\text{C}$ , the resistance increases to  $13.8 \Omega$ .

#### PART A

Find an equation in two unknowns representing **overall conservation of mass**:

$$\text{Total mass that goes into the alloy} = \text{Mass of alloy created}$$

Use the symbol **uppercase A** to represent the mass, in kg, of the 17% tin mixture, and use **uppercase B** to represent the mass, in kg, of the 9% tin mixture.

#### PART B

**Mass of tin** is also conserved, meaning that the amount of tin that goes into the alloy is the same as the amount of tin in the final alloy. Use the concentrations given to determine the **mass of tin** in each component of the mixture.

Use the symbol **uppercase A** to represent the mass, in kg, of the 17% tin mixture, and use **uppercase B** to represent the mass, in kg, of the 9% tin mixture.

- Mass of tin in **final 10.1% tin alloy**
- Mass of tin in **17% tin alloy** (in terms of A)
- Mass of tin in **9% tin alloy** (in terms of B)

## PART C

Determine the equation for conservation of mass of tin, using your answers to Part B above. Recall:

$$\text{Amount of tin that goes into the alloy} = \text{Amount of tin in the final alloy}$$

Use the symbol **uppercase A** to represent the mass, in kg, of the 17% tin mixture, and use **uppercase B** to represent the mass, in kg, of the 9% tin mixture. Enter your answer as an **equation**, including an "=" symbol.

## PART D

Solve the system of 2 equations in 2 unknowns found in Parts A and C. How much of each of the two modern alloys does the reenactor need to use to make the desired alloy?

Give your answers in kilograms, correct to at least three significant figures. Do **not** enter units in your answer.

### Additional Background Information:

Human prehistory (the time period pre-dating recorded history) is often subdivided into three ages, each of which is named for the primary tool-making technology of the era: the Stone Age, the Bronze Age, and the Iron Age. The Bronze Age in Europe covers a time period from 3200 BCE (Before Common Era) to 600 BCE. Tools and artifacts found from this time period are often made of copper alloys, especially bronze (made of copper and tin) and brass (made of copper and zinc). The use of bronze is indicative not only of the technological advances of the time, but is also indicative of trading, since copper and tin are not often able to be mined in the same geographical location.



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<sup>1</sup> Calame. "Epée, bronze, âge du bronze, vers 800 av. JC ???, musée des antiquités nationales, Saint-Germain-en-Laye." 7 November 2006. *Wikimedia Commons*. Web. 14 June 2013.

**Answers:**

A.  $A + B = 3.06$

B. Mass of tin in components:

i. Mass of tin in **final 10.1%** tin alloy = 0.30906

ii. Mass of tin in **17%** tin alloy (in terms of A) =  $0.17A$

iii. Mass of tin in **9%** tin alloy (in terms of B) =  $0.09B$

C.  $0.17A + 0.09B = 0.30906$

D. The reenactor needs to use 0.421 kg of the 17% tin alloy and 2.64 kg of the 9% tin alloy to create 3.06 kg of alloy that is 10.1% tin.



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## Resistance and Temperature

### Introduction:

The resistance ( $R$ ) of copper changes with temperature ( $T$ ), with resistance increasing as temperature increases. Within a wide temperature range (about  $-200^{\circ}\text{C}$  to  $500^{\circ}\text{C}$ ), this is a **linear relationship**. The exact relationship between the resistance and temperature of a wire depends on the cross-sectional area and length of the piece of wire.

### Problems: You Try It!

For one particular piece of copper wire, the resistance is  $13.2\ \Omega$  at a temperature of  $14^{\circ}\text{C}$ . When the temperature is increased to  $27^{\circ}\text{C}$ , the resistance increases to  $13.8\ \Omega$ .

#### PART A

Find the linear equation used to predict the resistance,  $R$ , in ohms ( $\Omega$ ) in the wire from its temperature,  $T$ , in degrees Celsius ( $^{\circ}\text{C}$ ).

Enter your answer **in slope-intercept form**. Be sure to use the correct variable, uppercase  $T$ , for temperature. Give constants correct to at least 3 significant figures.

#### PART B

Based on the equation found in Part A, identify the **slope** of the line, with appropriate units, **and** select the sentence below that best describes what that slope represents.

The slope represents:

- the change in resistance when the temperature increases by  $1^{\circ}\text{C}$ .
- the resistance when the temperature is  $0^{\circ}\text{C}$ .
- the temperature at which the resistance would be  $0\ \Omega$ .
- the change in temperature when the resistance increases by  $1\ \Omega$ .

#### PART C

Based on the equation found in Part A, identify the **R-intercept** of the line, with appropriate units, and select the sentence that best describes what that intercept represents.

The intercept represents:

- the temperature at which the resistance would be  $0\ \Omega$ .
- the change in resistance when the temperature increases by  $1^{\circ}\text{C}$ .
- the resistance when the temperature is  $0^{\circ}\text{C}$ .
- the change in temperature when the resistance increases by  $1\ \Omega$ .

#### PART D

Since the resistance varies linearly over a wide range of temperatures, the equation developed in Part A may be used to predict resistance at a particular temperature within that range, or the temperature at which the resistance will reach a particular value.

Based on this information, estimate the resistance in the wire at a temperature of  $49^{\circ}\text{C}$ .

Express your answer in ohms ( $\Omega$ ), correct to at least three significant figures. **Do not** enter units in your answer.

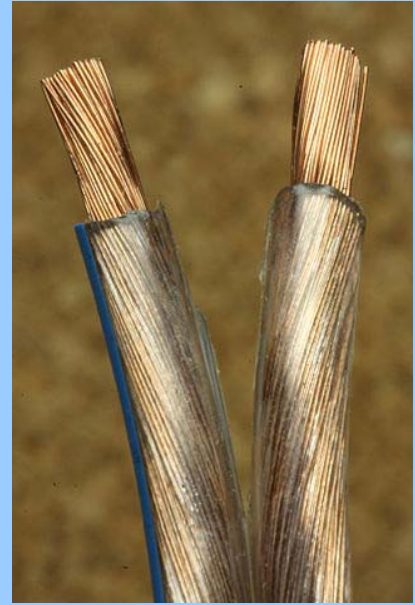
## PART E

Generally, we wish to reduce resistance in wire, as that reduces power losses as current flows through the wire. At what temperature would the resistance be reduced to  $9\ \Omega$ ?

Express your answer in degrees Celsius ( $^{\circ}\text{C}$ ), correct to at least three significant figures. If the temperature is negative, be sure to include a negative sign in your answer. Do **not** enter units in your answer.

### Additional Background Information:

Electricity is essential in our modern society; it is generated, transported, and used continuously. The most widely used material for the transfer of electricity these days is copper, which is not only an excellent conductor of electricity, but is also inexpensive and has high ductility (it is easily formed into wire). The resistance of a conductor (such as a copper wire) represents its opposition to current passing through it. The less resistance a conductor has, the more easily current passes through it. Resistance has units of ohms, represented by the symbol  $\Omega$ .



Copper wires<sup>1</sup>



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<sup>1</sup> RobertKuhlmann. "Lautsprecherkabel 1,5mm<sup>2</sup> in Makroaufnahme mit abisolierten Enden und Polmarkierung". 12 December 2011. Wikimedia Commons. Web. 22 May 2013.

**Answers:**

A.  $R = 0.0462T + 12.6$

B.  $m = 0.0462 \Omega/^\circ\text{C}$ . The slope represents change in resistance when the temperature increases by  $1^\circ\text{C}$ .

C.  $b = 12.6 \Omega$ . The  $R$ -intercept represents the resistance when the temperature is equal to  $0^\circ\text{C}$ .

D.  $14.8 \Omega$

E.  $-77.0^\circ\text{C}$



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## Earthquake Epicenter

### Introduction:

Earthquakes produce different types of waves which travel through the earth at different speeds (which vary slightly depending on local conditions). These waves types include **P-waves** (primary waves) and **S-waves** (secondary waves). P-waves travel **faster** near the surface of the earth than S-waves, so they arrive at any location near the earthquake first. The location of the epicenter of an earthquake can be found from information about these waves.

### Problems: You Try It!

Near a particular observatory, P-waves travel at a speed of 7.6 km/s and S-waves travel at a speed of 4.5 km/s. For one earthquake, the P-wave arrives at the observatory 19 seconds before the S-wave. Use this information in the problems below to determine the distance from the observatory to the center of the earthquake.

In the following questions, use the variable lowercase  $d$  to represent the distance from the epicenter of the earthquake to the observatory, and use the variable lowercase  $t$  to represent the time for the P-wave to get from the epicenter of the earthquake to the observatory.

#### PART A

Use the relationship between speed, distance, and time to develop an equation for  $t$  and  $d$  based on the information about the **P-wave** given above.

#### PART B

Use the relationship between speed, distance, and time to develop an equation for  $t$  and  $d$  based on the information about the **S-wave** given above.

#### PART C

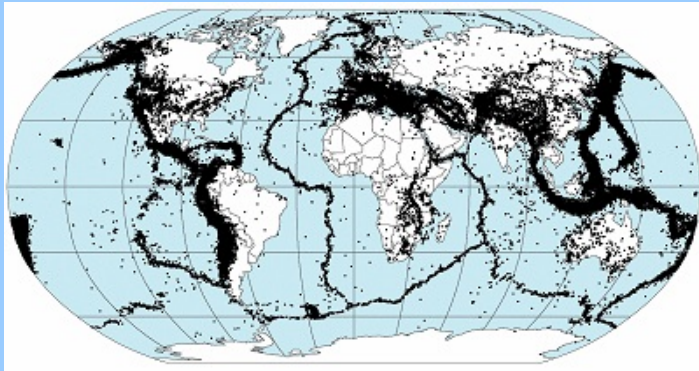
Solve the system of two equations in two unknowns from Part A and Part B to determine the following values.

Give your answers in the seconds (for time) and in kilometers (for distance), correct to at least three significant figures. Do **not** enter units in your answers.

- i. Time for the P-wave to travel from the epicenter of the earthquake to the observatory
- ii. Time for the S-wave to travel from the epicenter of the earthquake to the observatory
- iii. Distance from the epicenter of the earthquake to the observatory

### Additional Background Information:

Earthquakes can cause massive amounts of damage, including loss of life, and so are an important area of study. Determining the location of the epicenter of an earthquake (the point on the Earth's surface immediately above the point of origin of the earthquake) is useful in predicting damage from that earthquake and the behaviour of future earthquakes. One useful technique in doing this involves studying the different waves produced by an earthquake.



Map of Earthquake Epicenters from 1963 to 1998<sup>1</sup>

Earthquakes produce different types of waves which travel through the earth at different speeds (which vary slightly depending on local conditions). These waves types include **P-waves** and **S-waves**. P-waves are sometimes called primary waves or pressure waves, and S-waves are sometimes called secondary waves or shear waves. Since P-waves travel **faster** near the surface of the earth than S-waves, they arrive at any particular location first (hence "primary" waves and "secondary" waves).

These waves can be detected by equipment such as seismometers. The fact that P-waves and S-waves arrive at an observatory at different times can be used to determine **how far** from the observatory the **epicenter** of the earthquake is located. By combining such information from several observatories, the location of the epicenter of the earthquake can be determined.



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<sup>1</sup> Lowman, Paul and Brian Montgomery. "Quake epicenters: Preliminary Determination of Epicenters, 358,214 Events, 1963 - 1998." 3 June 2006. *Wikimedia Commons*. Web. 14 June 2013.



**Answers:**

A.  $d = 7.6t$

B.  $d = 4.5(t + 19)$

C. Answers:

i. 27.6 seconds

ii. 46.6 seconds

iii. 210 km



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